Short Papers

Finite-Element Analysis of Overmoded Waveguide Using Silvester's Algorithm

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***Abstract* — A general, high-order finite-element waveguide analysis program originated by Silvester [1], [2] has been used to analyze overmoded waveguides. The algorithm approximates arbitrarily shaped waveguides by triangular subsections and solves the Helmholtz equation subject to homogeneous Dirichlet or Neumann boundary conditions to obtain the eigenvalues (cutoff wavelengths) and the eigenvectors (scalar potentials). During these investigations of arbitrarily shaped overmoded waveguides, a computer program error was identified. This error resulted in incorrect higher-order-mode potential functions. As this algorithm has been rather widely disseminated, it is the purpose of this communication to inform users of a correction which yields the correct higher-order-mode potential functions.**

I. Introduction

The Silvester algorithm was obtained from Montgomery [3] and installed on the CYBER computer system at the Georgia Institute of Technology. Our purpose for the algorithm was to examine overmoded waveguides with arbitrary cross sections. In order to check the algorithm execution, rectangular and circular waveguides were used as test cases. During the numerical testing, it was observed that the lowest-order-mode potential was always computed correctly, as shown in Fig. 1. However, discontinuities in the potential were observed for the higher order modes, as shown in Fig. 2. These "spike" discontinuities preclude the computation of the higher-order-mode fields by differentiation of the TE- and TM-mode potential functions. Note that this differentiation can be performed numerically or that the polynomial basis functions can be differentiated directly to obtain an expression for the potential in terms of the polynomial coefficients. In our analysis, the direct differentiation was employed.

After an exhaustive investigation into the algorithm code, it was determined that a program error caused the incorrect results for the higher-order-mode potentials. Briefly, the method used by Silvester is to tridiagonalize the finite-element matrix for the eigenvalue problem given as

$$Ax = k^2 x. \tag{1}$$

As stated by Silvester, "this equation is solved by a Householder tridiagonalization. The eigenvalues are located by bisection, using Sturm sequences and the eigenvectors are computed by Wielandt iteration" [1]. After the tridiagonal matrix is solved, the eigenvectors of the original problem are constructed by multiplying the tridiagonal matrix by appropriate "rotation matrices," which are "remembered" during the tridiagonalization process. Unfortunately, during the first pass through this procedure, one of these matrices is overwritten which causes the subsequent (i.e., higher order) mode potential functions to be erroneous.

The original and corrected lines of code are given in Table I. When the correction is implemented, the higher order mode potentials are calculated correctly, as shown in Figs. 3 and 4 for the cases of rectangular and circular waveguide, respectively. The code can also be used to compute the potential functions for other guide shapes, as illustrated in Figs. 5 and 6, which present results for triangular and trapezoidal waveguides.

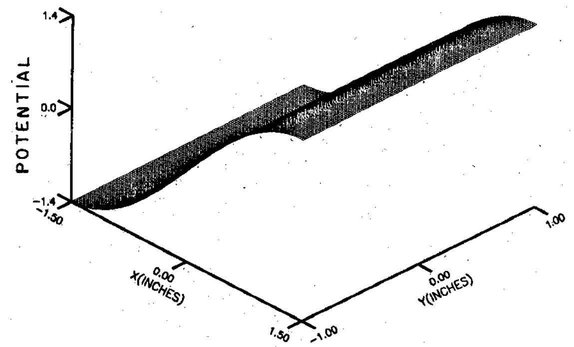

Fig. 1. Potential function for the first TE mode in a 3×2 rectangular waveguide using 18 triangular subsections and fourth-order polynomials.

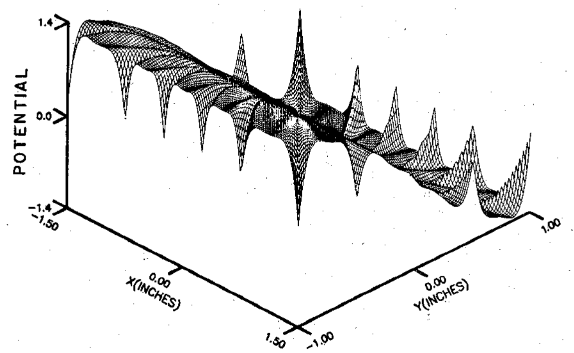

Fig. 2. Potential function for the second TE mode in a 3×2 rectangular waveguide using 18 triangular subsections and fourth-order polynomials.

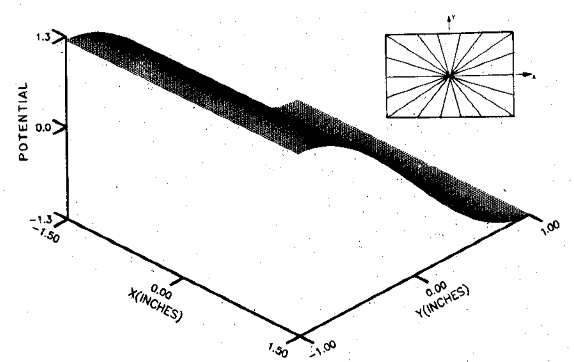

Fig. 3. Rectangular waveguide potential for the second TE mode.

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TABLE I
ORIGINAL AND MODIFIED CODE FOR SILVESTER'S PROGRAM

ORIGINAL CODE	MODIFIED CODE
`C     CALCULATE TRUE EIGENVECTOR FROM EIGENVECTOR OF A.`	`C     CALCULATE TRUE EIGENVECTOR FROM EIGENVECTOR OF A.`
`      CALL REVERS(A,SUBD,X,N)`	`      CALL REVERS(A,SUBD,X,N)`
`      CALL TRIMUL(X,T,X,N)`	`      CALL TRIMUL(X,T,X,N)`
`C`	`C`
`C     NORMALISE TO UNIT LENGTH.`	`C     NORMALISE TO UNIT LENGTH.`
`      CALL NORMAL(X,N,FLOAT(NPTS)/FLOAT(N))`	`      CALL NORMAL(X,N,FLOAT(NPTS)/FLOAT(N))`
`C`	`C`
`C     RESTORE STRUCK-OUT ENTRIES AND PRINT EIGENFUNCTION.`	`C     RESTORE STRUCK-OUT ENTRIES AND PRINT EIGENFUNCTION.`
`      IF(MOD(I,L).EQ.1)WRITE(6,107)`	`      IF(MOD(I,IP).EQ.1)WRITE(6,107)`
`      WRITE(6,103)I`	`      WRITE(6,103)I`
`      KK=1`	`      KK=1`
`      DO 12 J=1,NPTS`	`      DO 12 J=1,NPTS`
`      IF(MOD(KINDS(J),5).NE.0)GO TO 14`	`      IF(MOD(KINDS(J),5).NE.0)GO TO 14`
`      T(J)=X(KK)`	`      TST(J)=X(KK)`
`      GO TO 16`	`      GO TO 16`
`   14 T(J)=0.`	`   14 TST(J)=0.`
`      KK=KK-1`	`      KK=KK-1`
`   16 KK=KK+1`	`   16 KK=KK+1`
`   12 CONTINUE`	`   12 CONTINUE`
`      WRITE(6,106)(JX,T(JX),JX=1,NPTS)`	`      WRITE(6,106)(JX,TST(JX),JX=1,NPTS)`

The array TST(J) in the modified code must be dimensioned the same as the array T(J) in the original code.

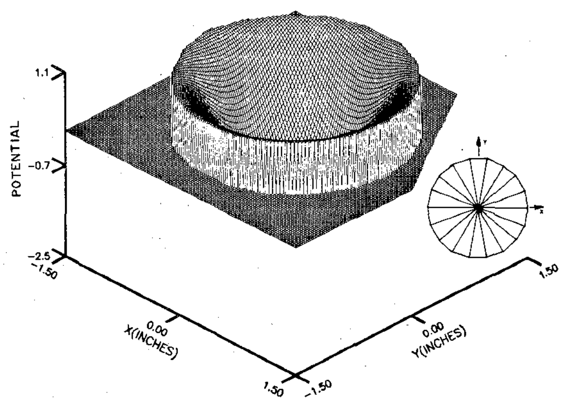

Fig. 4. Circular waveguide potential for the fifth TE mode.

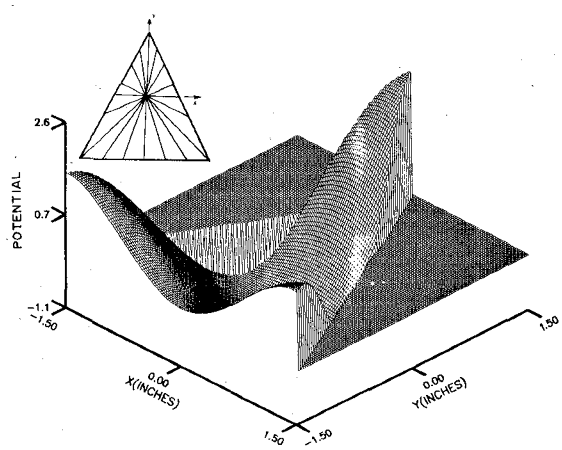

Fig. 5. Triangular waveguide potential for the third TE mode.

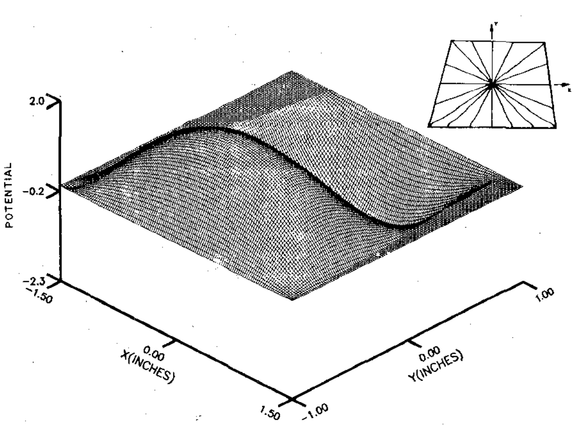

Fig. 6. Trapezoidal waveguide potential for the third TM mode.

II. CONCLUSIONS

The "General High-Order Finite-Element Waveguide Analysis Program" originated by Silvester [1], [2] has been used to compute the potential functions for the higher order modes in arbitrarily shaped waveguides. Upon correction of a program error, the code was found to be accurate and efficient. In addition, Silvester's assertion [1] that it is "desirable to use as few triangular subregions as the boundary shape will permit, with as high a degree of polynomial representation as feasible" was substantiated.

REFERENCES

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Dyadic Green's Functions for Integrated Electronic and Optical Circuits

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***Abstract*—Layered structures play an important role in both integrated microwave circuits and optical integrated circuits. Accurate prediction of device behavior requires evaluation of fields in the system. An increasingly used mathematical formulation relies on integral equations: the electric field in the device is expressed in terms of the device current integrated into an electric Green's function. Details of the development of the specialized Green's functions used by various researchers have not appeared in the literature. We present the development of general dyadic electric Green's functions for layered structures; this dyadic formulation allows extension of previous analyses to cases where currents are arbitrarily directed. The electric-field Green's dyads are found in terms of associated Hertzian potential Green's dyads, developed via Sommerfeld's classic method. Incidently, boundary conditions for electric Hertzian potential are utilized; these boundary conditions, which have been a source of confusion in the research community, are developed in full generality. The dyadic forms derived herein are reducible in special cases to the Green's functions used by other workers.**

I. INTRODUCTION

Layered dielectric structures, such as those depicted in Fig. 1, play an important role in both integrated electronic circuits and integrated optical circuits. In integrated electronics, conducting "devices" are affixed to a dielectric film layer which is deposited over a conducting ground plane. For integrated optical circuits, a dielectric waveguiding region is typically placed on top of a dielectric film layer; the film layer is, in turn, deposited on a

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